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# Gauge freedom for Gravitational Wave problems in tensor-scalar theories of gravity

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## Abstract

A specific choice of gauge is shown to imply a decoupling between the tensor and scalar components of Gravitational Radiation in the context of Brans-Dicke type theories of gravitation. The comparison of the predictions of these theories with those of General Relativity is thereby made straightforward.

# 1 Introduction

The fundamental postulate at the basis of any theory of gravitation is that of diffeomorphism invariance under general coordinate transformations. As a consequence, physically equivalent solutions to the ensuing dynamics fall into gauge equivalence classes, namely gauge orbits under the diffeomorphism group.

In General Relativity, the dynamics of the spacetime metric is solely determined from Einstein's equations. As just pointed out, a unique physical solution, associated to a unique geometry of the Universe, may be represented in terms of different functional solutions for the metric field, each corresponding to a different coordinate system, chosen to parametrize this geometry. Just like for Maxwell's equations for electromagnetism, any theory for gravity thus enjoys such a gauge freedom, which can be used to simplify the field equations and facilitate the interpretation of any phenomenon predicted by the theory. In particular, there exists some appropriate choice of gauge fixing when tackling the problem of Gravitational Waves.

Beyond General Relativity, a scalar gravitational sector has been largely suggested by the most promising theories for the description of fundamental interactions, that is, String Theories [1]. Among all tensor-scalar models, Brans-Dicke theories correspond to the simplest generalization of Einstein's theory including a scalar gravitational component. Obviously, the symmetry under general coordinate transformations remains valid in all consistent theories beyond General Relativity.

The aim of this letter is to discuss a choice of gauge fixing which seems to be particularly appropriate when tackling the problem of Gravitational Waves in tensor-scalar theories. But first, we briefly review the conventional gauge fixing choices in General Relativity and Brans-Dicke type theories.

## 2 Within the limits of General Relativity

In a weak field region, we can expand the metric field (in cartesian coordinates) in terms of a small perturbation  $h_{\mu\nu}$  around Minkowski's metric  $\eta_{\mu\nu}$ <sup>1</sup>, namely

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{2\kappa} h_{\mu\nu} , \quad (1)$$

where

$$\kappa = \frac{8\pi G}{c^4} . \quad (2)$$

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<sup>1</sup>Our signature convention for  $\eta_{\mu\nu}$  is  $(+, -, -, -)$ .

The linearized Einstein field equations for  $h_{\mu\nu}$  then read in any frame (indices being raised by means of  $\eta_{\mu\nu}$ ) [2]:

$$\square h_{\mu\nu} - h^\lambda_{\nu,\mu,\lambda} + h^\lambda_{\lambda,\mu,\nu} - h^\lambda_{\mu,\nu,\lambda} = -\sqrt{2\kappa} S_{\mu\nu} , \quad (3)$$

with

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T , \quad T = T^\lambda_\lambda . \quad (4)$$

$T_{\mu\nu}$  being the conserved stress-energy tensor for matter fields, computed on Minkowski's spacetime.

Consider then an infinitesimal coordinate transformation

$$x'^\mu = x^\mu + \varepsilon^\mu(x) , \quad (5)$$

with  $\varepsilon^\mu(x)$  of the same order of magnitude as the perturbation of the metric field. The metric being a second rank tensor, to first order the following transformation law for  $h_{\mu\nu}$  then applies,

$$\sqrt{2\kappa} h'_{\mu\nu} = \sqrt{2\kappa} h_{\mu\nu} - \partial_{(\mu} \varepsilon_{\nu)} , \quad (6)$$

which means that, if  $h_{\mu\nu}$  is a solution to the equations (3) in the original frame, then  $h'_{\mu\nu}$ , defined by (6), is another expression for the same physical solution in the frame obtained through the transformation (5).

The analysis of Gravitational Waves is always discussed in a class of "Harmonic" coordinate systems [2], defined by the four following conditions:

$$\partial_\mu \bar{h}^{\mu\nu} = 0 , \quad (7)$$

with

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h , \quad h = h^\lambda_\lambda . \quad (8)$$

These restrictions provide a simplified expression for the field equations (3):

$$\square \bar{h}_{\mu\nu} = -\sqrt{2\kappa} T_{\mu\nu} , \quad (9)$$

a clear indication for the existence of propagating wavelike solutions for the perturbation  $h_{\mu\nu}$ . However, a residual gauge freedom of the form (5) remains within this class of frames, constrained by the conditions

$$\square \varepsilon^\mu(x) = 0 , \quad (10)$$

whose general solution is thus of the form

$$\varepsilon^\mu(x) = i\varepsilon^\mu(k) e^{ik^\lambda x_\lambda} . \quad (11)$$

The choice of a specific frame thus requires 4 additional constraints, bringing the number of independent polarization states of the waves from 10 ( $h_{\mu\nu}$  being a symmetric tensor) down to 2. To understand the impact of a Gravitational Wave impinging on a system of masses,

it is far easier to consider this process in a comobile frame, that is, a coordinate system in which the grid of coordinates moves with the bodies [3]. In such a frame, one can express the gauge-invariant element of proper distance between two objects as

$$d\sigma^2 = (-\delta_{ij} + h_{ij}) dx^i dx^j , \quad (12)$$

where the coordinate difference  $dx^i$  is constant during the impact of the wave. One can thus easily understand how this distance gets modified by a Gravitational Wave.

These points having been recalled, let us now indicate why one usually speaks of a Transverse-Traceless (TT) gauge: the geodesic equation for a particle initially at rest reads, at  $t = 0$

$$\frac{d^2 x^i}{dt^2} = \frac{1}{2} \left( \frac{cdt}{d\tau} \right)^2 \left( 2h^i{}_{0,0} - h_{00}{}^{,i} \right) . \quad (13)$$

Hence, in order to keep fixed coordinate differences between bodies

$$\frac{dx^i}{dt} = 0 , \quad (14)$$

we need to restrict to configurations such that

$$h_{0\mu} = cst. \quad (15)$$

It is readily shown that the 8 conditions (7) and (15) are also implemented by

$$\partial_i h^{ij} = 0 , \quad (16)$$

$$h = 0 \quad (17)$$

and

$$h_{0\mu} = 0 , \quad (18)$$

that is, by virtue of (16) and (17), the waves are *transverse* and *traceless* in a comobile frame. This justifies the name "TT-gauge". Note that the physical effect of a Gravitational Wave on the proper distance (12) is actually transverse and traceless, as it may be readily seen by working in a comobile frame.

The point is that the interpretation of the wave impact would not be so direct in any other frame, where the geodesic equation cannot be reduced to (14). But, if one considers the existence of Gravitational Waves in a tensor-scalar theory, other considerations may lead to a different choice of gauge. A discussion of this specific issue is the purpose of the following section.

### 3 The case of Brans-Dicke theories

The action for Brans-Dicke theories reads:

$$S = -\frac{1}{16\pi} \int d^4x \sqrt{g} \left( \Phi R - \frac{\omega_{BD}}{\Phi} \Phi_{,\mu} \Phi^{,\mu} \right) + \int d^4x \mathcal{L}_{mat}(\Psi, g_{\mu\nu}) \quad (19)$$

where  $\Phi$  is the scalar gravitational component and  $\Psi$  stands for matter fields. For simplicity, we define the tensor and scalar perturbations, respectively on Minkowski's spacetime and around a constant expectation value for the scalar field:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{2\bar{\kappa}} h_{\mu\nu} , \quad (20)$$

and

$$\Phi = \Phi_0 \left( 1 + a_{BD} \sqrt{2\bar{\kappa}} \varphi \right) , \quad (21)$$

where

$$\bar{\kappa} = \frac{8\pi}{\Phi_0 c^4} \quad (22)$$

stands for the gravitational coupling, analogous to  $\kappa$  in General Relativity, while  $a_{BD}$  represents the scalar coupling to matter fields, defined as  $a_{BD}^2 = 1/(2\omega_{BD} + 3)$ . The linearization of the field equations for  $h_{\mu\nu}$  and  $\varphi$  [4] then implies

$$\begin{aligned} -\frac{1}{2} \left[ \square h_{\mu\nu} - h_{\nu,\mu,\lambda}^\lambda + h_{\lambda,\mu,\nu}^\lambda - h_{\mu,\nu,\lambda}^\lambda \right] - \frac{1}{2} \eta_{\mu\nu} \left[ -\square h + h^{\alpha\beta}_{,\alpha,\beta} \right] - \\ \left[ a_{BD} \varphi_{,\mu,\nu} - a_{BD} \varphi^{\cdot\lambda}_{,\lambda} \eta_{\mu\nu} \right] = \sqrt{\frac{\bar{\kappa}}{2}} T_{\mu\nu} \end{aligned} \quad (23)$$

and

$$\square \varphi = a_{BD} \sqrt{\frac{\bar{\kappa}}{2}} T . \quad (24)$$

Invariance under general coordinate transformations results once again in the transformation law (6) for the metric perturbation, while the scalar field remains invariant to first order.

Defining a symmetric tensor field [4]

$$\Theta_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h - a_{BD} \eta_{\mu\nu} \varphi \equiv \bar{h}_{\mu\nu} - a_{BD} \eta_{\mu\nu} \varphi \quad (25)$$

which thus transforms according to

$$\sqrt{2\bar{\kappa}} \Theta'_{\mu\nu} = \sqrt{2\bar{\kappa}} \Theta_{\mu\nu} - \partial_{(\mu} \varepsilon_{\nu)} + \eta_{\mu\nu} \partial^\lambda \varepsilon_\lambda , \quad (26)$$

one can impose the following gauge conditions

$$\partial^\mu \Theta_{\mu\nu} = 0 . \quad (27)$$

Once more, the field equations then simplify to ordinary wave equations, namely

$$-\frac{1}{2} \square \Theta_{\mu\nu} = \sqrt{\frac{\bar{\kappa}}{2}} T_{\mu\nu} \quad (28)$$

as well as

$$\square \varphi = a_{BD} \sqrt{\frac{\bar{\kappa}}{2}} T . \quad (29)$$

The same residual conditions remain as specified in (10), associated to a large residual choice of gauge conditions. Any particular choice leaves independent 3 (of the 11 initial) components

of the extended gravitational field. The following paragraphs are devoted to the interest of two specific choices, one of them being, to our knowledge, rather unconventional, but nevertheless extremely interesting.

The standard reference corresponds to the following set of conditions:

$$\partial^\mu \Theta_{\mu\nu} = 0, \partial^i \Theta_{ij} = 0 \text{ and } \Theta_{\mu\nu} = h_{\mu\nu} . \quad (30)$$

Even though the conditions

$$\Theta_{\mu\nu} = h_{\mu\nu} \quad (31)$$

seem natural in order to avoid a multiplication in the number of variables [4], they are not justified by themselves. These conditions are in fact equivalent to

$$\partial^\mu \Theta_{\mu\nu} = 0, \Theta_{\mu 0} = cst. \text{ and } \Theta_{\mu\nu} = h_{\mu\nu} , \quad (32)$$

which imply in particular the constraints

$$h_{\mu 0} = cst. , \quad (33)$$

and thereby define a comobile frame, whose usefulness need not be emphasized anymore. Whatever the gauge, one can easily understand that 2 of the 3 polarization states are helicity-2 ones (h=2), while the remaining one is scalar (h=0). However, the precise form of each polarization determines a specific perturbation in the proper distance between two masses in this comobile frame only. The general expression for the wave then reads (for a wave travelling in the z-direction):

$$\Theta_{\mu\nu}(x) = h_{\mu\nu}(x) =$$

$$h_+(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + h_\times(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + h_{scal}(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (34)$$

where  $h_+(x)$ ,  $h_\times(x)$  and  $h_{scal}(x)$  are the waveforms associated to the three different polarization states.

Let us now consider the problem from a different point of view: the presence of a scalar polarization of the waves, coupled to ordinary tensor waves, may complicate somewhat the calculation leading to any physical result related to the emission of gravitational radiation. One can thus consider, as a criterium for gauge fixing, **the possibility of decoupling the scalar and tensor parts** of the problem. This criterium may indeed be met by imposing another set of conditions:

$$\partial^\mu \Theta_{\mu\nu} = 0, \partial^i \Theta_{ij} = 0 \text{ and } \Theta = 0 . \quad (35)$$

This choice of frame defines a gauge in which the polarization states of  $\Theta_{\mu\nu}$  are transverse and traceless (call it, by analogy, the  $\Theta$ TT-gauge). For a wave travelling in the z-direction, the general form for  $\Theta_{\mu\nu}$  reads:

$$\Theta_{\mu\nu}(x) = \Theta_+(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \Theta_\times(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (36)$$

where  $\Theta_+(x)$  and  $\Theta_\times(x)$  are the amplitudes associated to the two polarization states of  $\Theta_{\mu\nu}$ . Both helicity-2 polarization states are totally independent of the scalar gravitational field. The variables  $\Theta_{\mu\nu}$  (h=2) and  $\varphi$  (h=0) have thus been rendered independent. Note that  $\Theta_{\mu\nu}(x)$  is now different from  $h_{\mu\nu}(x)$ ; the metric perturbation is still defined in terms of 3 polarization states: by virtue of the definition (25) for  $\Theta_{\mu\nu}$  and the condition  $\Theta = 0$ , we have indeed

$$h_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\Theta - a_{BD}\eta_{\mu\nu}\varphi = \Theta_{\mu\nu} - a_{BD}\eta_{\mu\nu}\varphi \quad (37)$$

which, in this frame, gives

$$h_{\mu\nu}(x) = h_+(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + h_\times(x) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + h_{scal}(x) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (38)$$

Obviously, two helicity-2 polarizations remain, while the third one is still scalar, but provides a conformally flat geometry to spacetime, which after all is natural when considering a massless scalar perturbation!

However, the great interest of this  $\Theta$ TT-gauge <sup>2</sup> is that, working with **independent variables**, it is possible to reconstruct a simple effective theory for the study of tensor-scalar waves. The corresponding action (the variation of which gives (28) and (29)) does indeed separate into two parts:

$$S = S_2 + S_0 \quad (39)$$

with

$$S_2 = \int d^4x \left[ \left( \frac{1}{4} \partial_\alpha \Theta^{\mu\nu} \partial^\alpha \Theta_{\mu\nu} - \sqrt{\frac{\bar{\kappa}}{2}} \Theta^{\mu\nu} T_{\mu\nu} \right) + \lambda^\nu \partial^\mu \Theta_{\mu\nu} + \lambda \Theta \right] \quad (40)$$

and

$$S_0 = \int d^4x \left[ \frac{1}{2} \partial_\alpha \varphi \partial^\alpha \varphi + a_{BD} \sqrt{\frac{\bar{\kappa}}{2}} \varphi T \right], \quad (41)$$

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<sup>2</sup>Note that in a recent paper [5] received after the present work was completed, the same choice of gauge fixing is being discussed. However, these authors do not seem to realize that the main interest of this choice resides in the decoupling of the tensor and scalar components.

Where  $\lambda^\nu$  and  $\lambda$  are Lagrange multipliers necessary to impose the conditions  $\partial^\mu \Theta_{\mu\nu} = 0$  and  $\Theta = 0$  only under which the equations (28) hold.

This new formalism should allow for a simplified analysis of Gravitational Wave phenomena within the context of tensor-scalar theories by making straightforward the transposition of theoretical results from General Relativity. By having decoupled the contributions of the tensor and scalar components of gravitational radiation, assessing the impact of the scalar gravitational component on physical observations should become feasible in a much more transparent and direct way.

Even though further study should provide more examples, we only consider, in the two last sections, the well-known problem of the energy emission rate by a gravitational source in the quadrupolar approximation. We eventually apply it to a determination of the scalar to tensor ratio for the energy emission rate in the case of the Hulse-Taylor Binary Pulsar (PSR1913+16).

## 4 Application...

Using the new effective action, a semi-classical calculation <sup>3</sup> for a quantized wave emitted by a classical source, immediately leads to mean emission rates for tensor and scalar radiation given by, respectively

$$\left\langle -\frac{dE_{(h=2)}}{dt} \right\rangle = \frac{1}{5c^5\Phi_0} \left\langle \frac{d^3}{dt^3} Q^{ij*}(t) \frac{d^3}{dt^3} Q_{ij}(t) - \frac{1}{3} \frac{d^3}{dt^3} Q_i^{i*}(t) \frac{d^3}{dt^3} Q_j^j(t) \right\rangle \quad (42)$$

and

$$\left\langle -\frac{dE_{(h=0)}}{dt} \right\rangle = \frac{a_{BD}^2}{2} \frac{1}{5c^5\Phi_0} \left\langle \frac{d^3}{dt^3} Q_i^{i*}(t) \frac{d^3}{dt^3} Q_j^j(t) + \frac{1}{3} \frac{d^3}{dt^3} Q^{ij*}(t) \frac{d^3}{dt^3} Q_{ij}(t) \right\rangle, \quad (43)$$

where the  $Q_{ij}(t)$  are the quadrupolar moments of the source

$$Q_{ij}(t) \equiv \frac{1}{c^2} \int d^3\vec{x}' T_{00}(\vec{x}', t) x'_i x'_j \equiv \int d^3\vec{x}' \rho(\vec{x}', t) x'_i x'_j, \quad (44)$$

while  $\rho(\vec{x}', t)$  is its matter density.

This clearly illustrates how theoretical results are straightforwardly extended beyond those of General Relativity. The helicity-2 energy emission rate given in (42) is totally equivalent to what General Relativity predicts <sup>4</sup>, while the additional contribution, given by (43), is shown to appear as a consequence of the existence of the scalar component of gravitation in Brans-Dicke theories.

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<sup>3</sup>This calculation is performed following the same scheme as the one used in ref. [6] within the specific context of General Relativity only, while we extend it independently to both tensor and scalar emissions as predicted in the context of Brans-Dicke theories.

<sup>4</sup>With the remaining difference that the gravitational coupling is different from the one of general relativity:  $\kappa \neq \bar{\kappa}$ .

One may thus define the mean expectation value for the scalar field  $\Phi_0$  to be  $\Phi_0 = \frac{1}{G}$ , keeping in mind that  $G$  differs from Newton's constant.



## 5 ...To the Binary Pulsar PSR1913+16

We may thus calculate now the correction induced by the scalar component of gravitation to the acceleration of the orbital motion for the Hulse-Taylor pulsar around its companion, this binary being, presently, our best laboratory for the study of Gravitational Waves in the radiative-strong-field regime. A somewhat long but direct enough calculation (extending the one done in ref. [7] for the case of General Relativity, i.e., for the tensor part) gives, respectively,

$$\left\langle -\frac{dE_{(h=2)}}{dt} \right\rangle = \frac{32}{5} \frac{G^4 M^3 \mu^2}{c^5 a^5} f(e) \quad (45)$$

and

$$\left\langle -\frac{dE_{(h=0)}}{dt} \right\rangle = \left( \frac{a_{BD}^2}{6} \right) \frac{32}{5} \frac{G^4 M^3 \mu^2}{c^5 a^5} g(e) , \quad (46)$$

where  $f(e)$  and  $g(e)$  are enhancement factors, relatively to the case of a circular orbit ( $e$  being the eccentricity of the orbit and  $a$  its semi-major axis), defined by

$$f(e) = \frac{(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4)}{(1 - e^2)^{7/2}} \quad (47)$$

and

$$g(e) = \frac{(1 + \frac{13}{4}e^2 + \frac{7}{16}e^4)}{(1 - e^2)^{7/2}} , \quad (48)$$

while  $M$  is the total mass of the binary and  $\mu$  its reduced mass.

These parameters have to be determined in terms of the predictions of the tensor-scalar theory. The correction, relatively to the predictions of General Relativity, thereby introduced in the tensor part of the radiation will be of the order of the scalar coupling  $a_{BD}^2$ , the value of which has to be inferior to  $10^{-3}$  according to solar system experiments. However, the ratio of the predicted rates of decrease in the orbital period only depends on the eccentricity of the orbit (assumed to be keplerian), the value of which is determined independently of the underlying theory. Hence, we may readily understand what is the impact of the scalar component thanks to this ratio (which is equal to the energy emission rates ratio, in the keplerian limit):

$$\frac{d}{dt}T_{(h=0)} / \frac{d}{dt}T_{(h=2)} = \frac{d}{dt}E_{(h=0)} / \frac{d}{dt}E_{(h=2)} = (a_{BD}^2/6) f(e) / g(e) . \quad (49)$$

For an eccentricity of the order of 0.6 and a scalar coupling fixed at  $a_{BD}^2 \simeq 10^{-3}$ , we find

$$\frac{d}{dt}T_{(h=0)} / \frac{d}{dt}T_{(h=2)} \big|_{1913+16} \preccurlyeq 2 \cdot 10^{-4} . \quad (50)$$

The observed rate of decrease in the orbital period of the binary ([8],[9]), consistent with the predictions of General Relativity is given by

$$\frac{d}{dt}T_{obs} \big|_{1913+16} = -2.422(6) \cdot 10^{-12} s/s , \quad (51)$$

where the figure in parentheses represents a  $1\sigma$  uncertainty in the last quoted digit. All corrections applied, the precision of this measurement reaches a value of  $3.5 \cdot 10^{-3}$ . Hence, the weakness of the scalar corrections just introduced suggests that it is not possible to make any conclusion relative to a potential scalar gravitational emission, and, thereby, discriminate between General Relativity and tensor-scalar models through the analysis of the Hulse-Taylor binary system.

Note that this issue has already been studied in much more details in ref. ([9],[10],[11]) where it has been shown indeed, that the predictions of Brans-Dicke theories fall, as well as those of General Relativity, within experimental error bars. However, the point we wanted to emphasize here is that the formalism defined above may be applied to render more transparent some results on this subject.

## 6 Conclusion

All theories of gravitation enjoy the gauge freedom associated to the arbitrariness in the choice of coordinate system locally parametrizing the spacetime manifold. The main point of this letter is to emphasize the interest of *different* choices for *different* purposes, particularly in the context of tensor-scalar theories. On the one hand, a comobile frame is always (whatever the theory) most appropriate when exploring the impact of a Gravitational Wave impinging on a system of masses and, specifically, to analyze the reaction of a Gravitational Wave detector. On the other hand, the choice of the so-called  $\Theta$ TT-gauge allows the decoupling of the tensor and scalar components of radiation, which is of great interest when comparing predictions from tensor-scalar theories and General Relativity. This is shown explicitly in the last two sections, in which our formalism is applied to the assessment of the scalar contribution to the decrease rate in the orbital period for a binary system.

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